1. Let ABC be a triangle. The angle bisectors of $\angle ABC$ and $\angle ACB$ intersect at D. If $\angle BAC = 80^\circ$, what are all possible values for $\angle BDC$?

2. $ABCDEF$ is a regular hexagon. Let R be the overlap between $\triangle ACE$ and $\triangle BDF$. What is the area of R divided by the area of $ABCDEF$?

3. Let M be on segment BC of $\triangle ABC$ so that $AM = 3$, $BM = 4$, and $CM = 5$. Find the largest possible area of $\triangle ABC$.

4. Let $ABCD$ be a rectangle. Circles C_1 and C_2 are externally tangent to each other. Furthermore, C_1 is tangent to AB and AD, and C_2 is tangent to CB and CD. If $AB = 18$ and $BC = 25$, then find the sum of the radii of the circles.

5. Let $A = (1, 0)$, $B = (0, 1)$, and $C = (0, 0)$. There are three distinct points, P, Q, R, such that $\{A, B, C, P\}$, $\{A, B, C, Q\}$, $\{A, B, C, R\}$ are all parallelograms (vertices unordered). Find the area of $\triangle PQR$.

6. Let $\mathcal{C}$ be the sphere $x^2 + y^2 + (z - 1)^2 = 1$. Point P on $\mathcal{C}$ is $(0, 0, 2)$. Let $Q = (14, 5, 0)$. If PQ intersects $\mathcal{C}$ again at Q', then find the length PQ'.

7. Define $A = (1, 0, 0)$, $B = (0, 1, 0)$, and $\mathcal{P}$ as the set of all points (x, y, z) such that $x+y+z = 0$. Let P be the point on $\mathcal{P}$ such that $d = AP + PB$ is minimized. Find d^2.

8. Suppose that $A = \left(\frac{1}{2}, \sqrt{3}\right)$. Suppose that B, C, D are chosen on the ellipse $x^2 + (y/2)^2 = 1$ such that the area of $ABCD$ is maximized. Assume that A, B, C, D lie on the ellipse going counterclockwise. What are all possible values of B?

9. Let ABC be a triangle. Suppose that a circle with diameter BC intersects segments CA, AB at E, F, respectively. Let D be the midpoint of BC. Suppose that AD intersects EF at X. If $AB = \sqrt{9}$, $AC = \sqrt{10}$, and $BC = \sqrt{11}$, what is $\frac{EX}{XF}$?

10. Let ABC be a triangle with points E, F on CA, AB, respectively. Circle C_1 passes through E, F and is tangent to segment BC at D. Suppose that $AE = AF = EF = 3$, $BF = 1$, and $CE = 2$. What is $\frac{ED^2}{FD^2}$?

P1. Suppose that circles C_1 and C_2 intersect at X and Y. Let A, B be on C_1, C_2, respectively, such that A, X, B lie on a line in that order. Let A, C be on C_1, C_2, respectively, such that A, Y, C lie on a line in that order. Let A', B', C' be another similarly defined triangle with $A \neq A'$. Prove that $BB' = CC'$. (You must include a diagram with your solution).

P2. Suppose that fixed circle C_1 with radius $a > 0$ is tangent to the fixed line l at A. Variable circle C_2, with center X, is externally tangent to C_1 at $B \neq A$ and l at C. Prove that the set of all X is a parabola minus a point.