1. Compute the maximum real value of a for which there is an integer b such that $\frac{ab^2}{a+2b} = 2019$. Compute the maximum possible value of a.

   Answer: 30285

   Solution: We can solve the equation for a, obtaining $a = \frac{4038b}{b^2-2019}$. This is maximized when b^2 is as close to 2019 while still exceeding it. We let $b = 45$ and as a result $a = 30285$.

2. If P is a function such that $P(2x) = 2^{-3}P(x) + 1$, find $P(0)$.

   Answer: $\mathbf{\frac{8}{7}}$ **or** $\mathbf{1\frac{1}{7}}$

   Solution: Plugging in $x = 0$, we obtain

   $$P(0)(1 - 2^{-3}) = 1,$$

   so $P(0) = \boxed{\frac{8}{7}}$.

3. There are two equilateral triangles with a vertex at $(0, 1)$, with another vertex on the line $y = x + 1$ and with the final vertex on the parabola $y = x^2 + 1$. Find the area of the larger of the two triangles.

   Answer: $\mathbf{45 + 26\sqrt{3}}$

   Solution: We can shift all three vertices down one unit with no change to the areas of the triangle. We then have a vertex at $(0, 0)$, another vertex at (a, a), and a third vertex at (b, b^2). Representing as polar coordinates, they are at $0, (a + ai)$, and $b + b^2i$. We know that either $(a + ai)(\frac{1}{2} + \frac{\sqrt{3}}{2}i) = b + b^2i$, or $(b + b^2i)(\frac{1}{2} + \frac{\sqrt{3}}{2}i) = a + ai$. Solving both, we see that our maximal triangle has solution where $a = 5 + 3\sqrt{3}$ (which comes from the first equation; the second equation only has the solution $a = b = 0$), which gives us an area of $45 + 26\sqrt{3}$.